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Compotonization:

If the evolution of the spectrum of a source is primarily determined by Compton scattering, this process is often referred to as "Comptonization". The plasma must be rarefied in this case so that other radiation processes such as bremsstrahlung do not contribute extra photons into the system.

If the plasma is hot, then the exchange of energy per collision is greater if the matter is hotter than the radiation. Examples of sources in which such conditions are found include the hot gas in the vicinity of binary X-ray sources, the hot plasma in AGNs, the hot intergalactic gas in galaxy clusters, and the primordial plasma in the early universe before recombination.

Here we try to build a simple picture of the Comptonization process. We will restrict our discussion to the regime where $kT \ll m_e c^2$ and

$\hbar\omega \ll m_e c^2$. First, recall the expression for the energy transferred to stationary electrons when $\hbar\omega \ll m_e c^2$:

$$\frac{\Delta E}{E} = \frac{\hbar\omega}{m_e c^2}(1-\cos\theta) \qquad (\theta: \text{scattering angle})$$

The average energy loss of the photon in this case is:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{\hbar\omega}{m_e c^2}$$

Next, the low-energy limit of the energy loss rate of high-energy electrons in Inverse Compton scattering off low-energy photons is:

$$\frac{dE}{dt} = \frac{4}{3}\sigma_T c \left(\frac{v}{c}\right)^2 U_{rad}$$

This energy is gained by the photon, with the average energy gain per collision given by:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3}\left(\frac{v}{c}\right)^2$$

If the electrons have a thermal distribution, we have:

$$\frac{1}{2} m_e \langle v^2 \rangle = \frac{3}{2} kT$$

As a result, the net energy change of the photon in a Compton collision is:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{-\hbar\omega_0}{m_e c^2} + \frac{4kT}{m_e c^2}$$

If $4kT > \hbar\omega_0$, energy is transferred to the photon, while if $\hbar\omega_0 < kT$ energy is transferred to the electrons. There is no net energy transfer if $\hbar\omega_0 = 4kT$.

We are primarily concerned with the case where the electrons are hotter than the photons. The fractional increase in energy is $\frac{4kT}{m_e c^2}$ per collision in this case. If the region has electron density n_e and size l, the optical depth for Thomson scattering is $\tau_e = n_e \sigma_T l$. If $\tau_e \gg 1$, the photons undergo a random walk in scaping from the region. The net distance travelled by the photon is $N^{\frac{1}{2}} \lambda_e$, where N is the number of scatterings and $\lambda_e = (n_e \sigma_T)^{-1}$ is the mean free path of the photon.

The number of scatterings is:

$$N = \left(\frac{l}{\lambda_e}\right)^2 = \tau_e^2$$

If $\tau_e \gg 1$, the number of scatterings is so large that the photon spectrum will be distorted by Compton scattering. The condition for significant distortion is given by $y \gtrsim 1$, where:

$$y = \frac{kT}{m_e c^2} \max(\tau_e, \tau_e^2)$$

The normal condition for Comptonization to change the spectrum significantly is:

$$y = \frac{kT}{m_e c^2} \tau_e^2 \gtrsim \frac{1}{4}$$

After N scatterings the energy of the photon relative to its initial energy is:

$$\frac{E'}{E} = \left(1 + \frac{4kT}{m_e c^2}\right)^N$$

Since $4kT \ll m_e c^2$, we can write:

$$\ln \frac{4kT}{m_e c^2} \simeq \exp\left(\frac{4kT}{m_e c^2}\right) \Rightarrow \frac{E'}{E} \simeq \exp(4y)$$

If the photons are heated and acquire an energy $\hbar\omega \simeq 4kT$, there will be no net energy transfer further. The optical depth necessary for this to occur is found by setting $E \stackrel{!}{=} 4kT_e$:

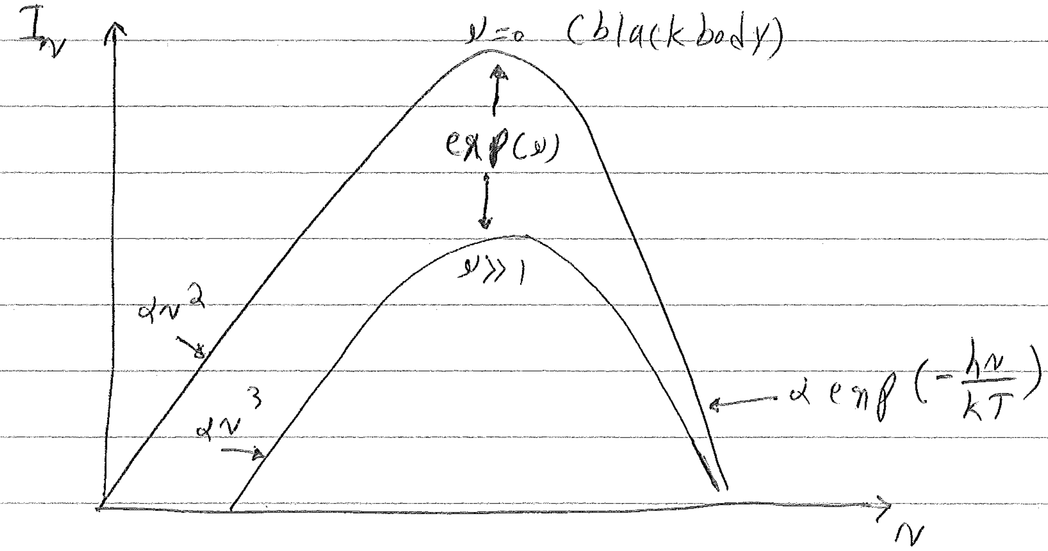
$$4kT = \hbar\omega_0 \exp(4y) \Rightarrow \frac{4kT}{\hbar\omega_0} = \exp\left[4\left(\frac{kT}{m_e c^2}\right)\tau_T^2\right] \Rightarrow$$

$$\tau_T = \left[\frac{m_e c^2}{4kT} \ln\left(\frac{4kT}{\hbar\omega_0}\right)\right]^{\frac{1}{2}}$$

If the optical depth of the medium is greater than this, the photon distribution approaches its equilibrium value determined entirely by Compton scattering. The equilibrium spectrum is given by the Bose-Einstein distribution;

$$f_{(\nu)} = \frac{1}{\exp\left(\frac{h\nu}{kT} + \mu\right) - 1} \qquad (f: \text{occupation number})$$

Here μ is the chemical potential of photons. If $\mu \ll 1$, then the distribution will lead to the blackbody spectrum. In the opposite limit $\mu \gg 1$, the spectrum looks like a blackbody at high frequencies (i.e., in the Wien part), while it is modified in the Rayleigh-Jens part at low frequencies. An illustration of the spectra is given in the following figure:



The question is how to describe the evolution of the spectrum toward equilibrium. In the non-relativistic limit, the equation that describes this evolution is called the "Kompaneets Equation". This equations takes account of the interchange of energy between photons and electrons in both directions, as well as induced effects that become important when the occupation number f is larger than 1.

The Kompaneets equation is:

$$\frac{\partial f}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x}\left[x^4\left(f + f^2 + \frac{\partial f}{\partial x}\right)\right]$$

Here:

$$dy = \frac{kT}{m_e c^2}\,\sigma_T\, n_e\, dl \quad , \qquad x \equiv \frac{h\nu}{kT}$$

The first term on the right-hand side $\propto f$ represents the cooling of the photon by the recoil effect. The second term $\propto f^2$ describes the effects of induced Compton scattering and becomes important when $n \gtrsim O(1)$. The last term $\propto \frac{\partial f}{\partial x}$ represents the diffusion of photons along the frequency as a result of (induced) Compton scattering. It is easy to show that the right-hand side vanishes in thermal equilibrium, i.e., when $f = [\exp(x+\mu) - 1]^{-1}$.

In general, the solutions to the Kompaneets equation have to be found numerically, but some useful limiting cases exist.

Sunyaev-Zeldovich Effect:

A very important application of the Kompaneets equation is in describing distortions of the spectrum of the Cosmic Microwave Background (CMB) radiation. This happens if the CMB photons are scattered by hot electrons in regions of very hot ionized gas as they propagate to the earth. The largest effect is expected in the direction of those rich galaxy clusters that possess large amounts of hot gas, for example the Perseus cluster. The effect is known as the Sunyaev-Zeldovich effect. The Sunyaev-Zeldovich effect results in depletion of the Rayleigh-Jeans part of the blackbody spectrum and population of the Wien part. The boundary between the parts is given by the frequency $h\nu = 4\,kT$, for which there is no net energy transfer. The change in the brightness temperature of the

CMB radiation in the Rayleigh-Jeans region is proportional to $n_e T$, with n_e being the number density of the electrons in the cluster. Therefore it provides an estimate of the pressure of the hot gas.

The Sunyaev-Zeldovich effect has a very important cosmological application. The temperature of the hot intergalactic gas in the cluster can be determined from the shape of the bremsstrahlung spectrum as pointed out before. For example, in the case of the Perseus cluster one finds $T = 7.5 \times 10^7$ K (corresponding to an energy of 6.5 keV). One can use this combined with the Sunyaev-Zeldovich effect to find the electron density and the physical size of the emitting gas cloud. By measuring its angular extent, the distance to the cluster can be measured. If the redshift of the cluster is known, one can measure the Hubble expansion rate at the present time.

The change in the brightness temperature of the CMB in the Rayleigh-Jeans part of the spectrum in the direction of reach clusters of galaxies is $O(mK)$. This is compatible with the average energy gain by the photons $\alpha \frac{kT}{m_e c^2}$ in a Compton collision for temperatures $T \sim 10^7 - 10^8$ K. It also underlines the fact that Compton emission is more important for non-thermal sources.